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Research Article

ON THE CLASSIFICATION OF PAIRS OF DISCRETE MEASURABLE PARTITIONS OF TYPE II

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ABSTRACT

In this paper, we consider the problem of classifying maximal approximatively finite pairs of discrete measurable partitions of type II Lebesgue space and construct a complete invariant for such pairs of partitions.

KEYWORDS

Lebesgue space, discrete measurable partitions, a one-to-one and mutually measurable mapping.

INTRODUCTION

A partition of a Lebesgue space is called discrete if it is a trajectory partition of some finite or countable group of automorphisms. By an automorphism we mean a one-to-one and mutually measurable mapping of a Lebesgue space onto itself, taking a zero-measure set to a zero-measure set. The Lebesgue space in this note is a space with σ -finite Lebesgue measure that does not contain points of positive measure. We will consider pairs of discrete measurable

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partitions $\{\xi_1, \xi_2, X\}$ of the space (X, F, m) satisfying the condition $\xi_1 \vee \xi_2 = \varepsilon$. Denote by $\xi_1 \wedge \xi_2$ a partition defined as follows: two points x, y belong to the same element $\xi_1 \wedge \xi_2$ if and only if there exists a finite sequence of points $x, x_1, x_2, ..., x_n, y$ for which every two neighboring points belong to the same element ξ_1 or ξ_2. Let $A \in F$, $mA > 0$ A pair of partitions $\{\xi_1 \cap A, \xi_2 \cap A, A\}$ is called a part of a pair $\{\xi_1, \xi_2, X\}$ if the following conditions are met:

1) Every element ξ_1 and every element ξ_2 has a non-empty intersection with *A.*
2) $(\xi_1 \wedge \xi_2) \cap A = (\xi_1 \cap A) \wedge (\xi_2 \cap A)$

A pair of discrete measurable partitions $\{\xi_1, \xi_2, X\}$ will be called an extension of the pair $\{\xi_1^1, \xi_2^1, X^1\}$ if the pair $\{\xi_1^1, \xi_2^1, X^1\}$ is isomorphic to some part of the pair $\{\xi_1, \xi_2, X\}$. A pair of discrete measurable partitions is said to be maximal if it has no extensions that are not isomorphic with it. Partition type $\xi_1 \wedge \xi_2 / \xi_2$ is called the type of the pair $\{\xi_1, \xi_2, X\}$. A pair $\{\xi_1, \xi_2, X\}$ is called ergodic if the partitions $\xi_1 \wedge \xi_2 / \xi_2$ are ergodic.

The concept of a maximal pair of discrete measurable partitions was defined in [1]. It was also shown there that the problem of describing pairs of discrete measurable partitions is largely reduced to describing maximal pairs. In [2] , [3-19] maximal pairs of type II $_1$ are described . The purpose of this paper is to generalize the results of [4] to the nonergodic case. If ζ is some partition of the space F $(X, , m)$, then by [ζ] we denote the group of all automorphisms that leave the partition ζ fixed. If ζ is a discrete partition of type II, then by m^ς we denote the invariant measure with respect to [ζ].Let be an ergodic pair of discrete type $\{\xi_1, \xi_2, X\}$ II measurable partitions of the space (X, F, m). If A and B are maximal one-layer sets with respect to ξ_1 (or ξ_2), then it is clear that

$$m^{\xi_1 \wedge \xi_2}(A) = m^{\xi_1 \wedge \xi_2}(B),$$

Denote by λ_1 $m^{\xi_1 \wedge \xi_2}$ – measure of the maximum one-layer set with respect to ξ_1, and through λ_2 $m^{\xi_1 \wedge \xi_2}$ – measure of the maximum one-layer set with respect to ξ_2.

Obviously, a pair of $\{\xi_1, \xi_2, X\}$ type II has type II $_\infty$ if and only if $\lambda_2 = \infty$.

For an ergodic pair of discrete measurable partitions $\{\xi_1, \xi_2, X\}$ of type II $_\infty$, we denote

$$\lambda_{\xi_1, \xi_2} = \begin{cases} 0, & \lambda_1 < \infty \\ 1, & \lambda_1 = \infty \end{cases}$$

The following theorem was proved in [21-47].

Theorem. Let $\{\xi_1, \xi_2, X\}$, $\{\xi_1^1, \xi_2^{1.}, X_1\}$ be ergodic maximal approximatively finite pairs of discrete measurable partitions of type II $_\infty$ of spaces (X, F, m), (X_1, F_1, m_1) respectively. Pairs of partitions

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$\{\xi_1, \xi_2, X\}$ and $\{\xi_1^1, \xi_2^{1.}, X_1\}$ are isomorphic if and only if $\lambda_{\xi_1,\xi_2} = \lambda_{\xi_1^1,\xi_2^1}$.[48-54].

Let $\{\xi_1, \xi_2, X\}$ a pair of discrete measurable partitions of type II ∞ of the space (X, F, m). For a discrete partition ζ, we denote $\theta(\varsigma)$ the measurable shell ζ, and by *C(x)* the partition element $\theta(\xi_1 \wedge \xi_2)$ containing x.

Let

$$E^{(0)}_{\xi_1,\xi_2} = \{x \in X : \lambda_{\xi_1 \cap C(x), \xi_2 \cap C(x)} = 0\},$$

$$E^{(1)}_{\xi_1,\xi_2} = \{x \in X : \lambda_{\xi_1 \cap C(x), \xi_2 \cap C(x)} = 1\}.$$

It is obvious that $E^{(0)}_{\xi_1,\xi_2}$, $E^{(1)}_{\xi_1,\xi_2}$ are measurable, Denote

$$\delta(M) = \begin{cases} 0, & mM = 0 \\ 1, & mM > 0 \end{cases}$$

$$\bar{\delta}_{\xi_1,\xi_2} = (\delta(E^{(0)}_{\xi_1,\xi_2}), \delta(E^{(1)}_{\xi_1,\xi_2})).$$

The following theorem holds:

Theorem. Let $\{\xi_1, \xi_2, X\}$ both $\{\xi_1^1, \xi_2^{1.}, X_1\}$ maximal approximatively finite pairs of discrete measurable partitions of type II ∞ spaces (X, F, m), (X_1, F_1, m_1) and quotient spaces $X/\theta(\xi_1 \wedge \xi_2)$, $X_1/\theta(\xi_1^1 \wedge \xi_2^1)$ be continuous Lebesgue spaces . Pairs of partitions $\{\xi_1, \xi_2, X\}$ and $\{\xi_1^1, \xi_2^{1.}, X_1\}$ are isomorphic if and only if

$$\bar{\delta}_{\xi_1,\xi_2} = \bar{\delta}_{\xi_1^1,\xi_2^1}.$$

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