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Research Article

NEW RULE FOR FINDING THE AREA OF A RIGHT-ANGLED TRIANGLE

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ABSTRACT

In the current article, the researcher presents a new method for finding the area of a right-angled triangle through the hypotenuse and one of the two acute angles in the triangle. It is used in the case of knowing both, and this rule is considered by the researcher as a qualitative leap in the world of mathematics that helps students, mathematicians, and those interested in mathematics to calculate the area of a right-angled triangle when the data is the length of the hypotenuse and the measure of any of the two acute angles in it.

KEYWORDS

Triangle, right-angled triangle, area of right-angled triangle, new method for finding the area of a right-angled triangle.

Mathematics Subject Classification (2020) –79E50, 97E10.

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INTRODUCTION

The triangle is one of the important mathematical shapes in the world of mathematics. We often see this geometric shape in our daily life, which leads us to scrutinize this shape and search for its basic properties and qualities. This area, however, at this accelerated time, we need additions that facilitate the process of calculating the area of a triangle to add to the mathematics that are the basic components of applied science.

The triangle varies in terms of its sides and angles, and each of these types has characteristics, relationships, and rules that distinguish it from others in terms of calculating the area, finding the perimeter, and many other applications.

Our focus in this article is on the right-angled triangle, specifically presenting a new method for calculating its area and its mathematical proof supported by practical examples.

Preliminary

1. A triangle is referred to as a regular polygon that has three sides. The unique property of a triangle is that the sum of any two sides of the triangle is always greater than the measure of the third side of the triangle. In simpler words, a triangle is just a closed figure of three sides that has a sum of its angles equal to 180 degree. Each shape of the triangle is classified based on the angle made by the two adjacent sides of the particular triangle: Acute angle triangle, Right angle triangle, Obtuse angle triangle.

2. Right Angled Triangle

2.1 Definition:

A right triangle is a triangle in which one angle is a right angle (90-degree angle). The relation between the sides and other angles of the right triangle is the basis for trigonometry [1] . Right Angled Triangle is the geometrical shape, it has 3 sides: Base, Hypotenuse, Height, and the angle between the base and the height of the triangle is always 90 degrees, The side opposite to the right angle is called the hypotenuse, and the two sides of right angle called base and high of the right-angle triangle [2].

2.2 Properties of Right - Angled Triangle

The right- angled triangle has all follow properties:

1. One angle always measure 90-degree.
2. The longest side of the right-angle triangle is called the hypotenuse of triangle.
3. Sum of two interior acute angles is always 90-degree.
4. The sides adjacent to the 90 degree-right angle in the triangle are known the base and perpendicular of the triangle.
5. The perpendicular draw from the right angle and join it to the hypotenuse, always make three similar kinds of triangles.

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6. The circle draw along the three vertices of the triangle, the radius of the circle drawn always equal to half the actual length of the hypotenuse.
7. The two angles, other the right angle in a right-angle triangle, are always acute angles [3].

Pythagorean theorem

In any right-angled triangle, the area of the square built on the hypotenuse of a right-angled triangle (the side opposite the right angle) is equal to the area of the squares built on both sides of the right angle (the two sides that meet at a right angle) [4] .

From the figure1,

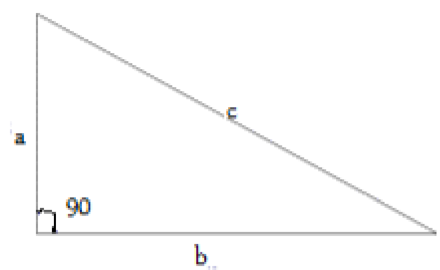

Figure 1 right-angled triangle

the following equation can be formed

$c = \sqrt{a2 + b2}$ ……. (1)

2.3 Area

As with any triangle, the area is equal to one half the base multiplied by the corresponding height. In a right triangle, if one leg is taken as the base then the other is height, so the area of a right triangle is one half the product of the two legs [5]. As a formula the area D is

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$$D = \frac{A.B}{2} \quad .... \quad (2)$$

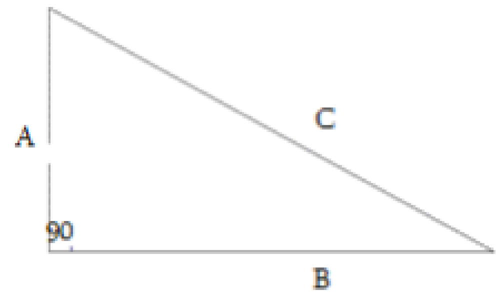

Figure 2 right-angled triangle

Altitudes

Altitude of a right triangle

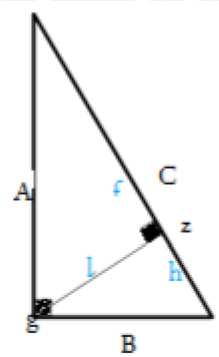

Figure 3 right-angled triangle

If a height is drawn from the vertex of the right angle to the hypotenuse of a right-angled triangle, the triangle is divided into two smaller triangles, and these two triangles are similar to the triangle that contains them, and they are similar to each other. Here we note the following:

- The height drawn to the hypotenuse of a right triangle is the mean of the proportionality of the two parts of the hypotenuse.
- Each side of a right angle in a triangle is the average proportionality between the

hypotenuse portion adjacent to the side and the hypotenuse of the triangle [6].

New rule to finding the area of right-angle triangle

After the researcher studied the right-angled triangle, and studied the hypotenuse as a basis for calculating the area of a right-angled triangle and using mathematical relationships, the researcher reached the following rule in calculating the area of a right-angled triangle in terms of the hypotenuse and the measurement of any of the two acute angles in it. This rule is:

$$Area\ of\ right\ triangle = \frac{\sin 2x \cdot length\ of\ hypotenuse\ the\ right\ angle\ triangle}{4} \quad (3)$$

when x any acute angle of the right triangle angle.

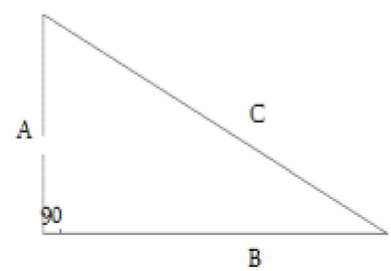

Figure 4 right-angled triangle

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Proof

From the triangle in figure 4 we see

$$\sin x + \cos x = \frac{A}{C} + \frac{B}{C} = \frac{A+B}{C}$$

Square two side we get $(\sin x + \cos x)^2 = (\frac{A+B}{C})^2$

$$\sin^2 x + 2 \sin x \cos x + \cos^2 x = \frac{(A+B)^2}{C^2}$$

$$1 + \sin 2x = \frac{A^2+2AB+B^2}{C^2} = \frac{2AB+A^2+B^2}{(\sqrt{A^2+B^2})^2}$$

$$1 + \sin 2x = \frac{2AB+A^2+B^2}{A^2+B^2} = \frac{2AB}{A^2+B^2} + \frac{A^2+B^2}{A^2+B^2}$$

$$1 + \sin 2x = \frac{2AB}{C^2} + 1$$

$$\sin 2x = \frac{2(2Area\ of\ right\ triangle\ )}{c2}$$

$$Area\ of\ right\ triangle = \frac{\sin 2x \,.(\,length\ of\ hypotenuse\ the\ \ right\ angle\ triangle)2}{4} \quad \ldots\ldots \quad (3)$$

when x any acute angle of the right triangle angle

example: to applicate the two formula to find the area of right- angle triangle.

Table No. (1) includes a set of practical examples, including five right-angled examples with different side lengths to emphasize the rule of the area of a right-angled triangle in terms of the length of the hypotenuse and the acute angle in it.

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table (1) calculate the area use formula1 and formula 3

base	high	hypotenuse	area when use formula (1)	hypotenuse square	sin2x of acute angle in the right-angled triangle	area when use formula (3) that reach from researcher
4	3	5	$\frac{1}{2}(4)(3)=6$	25	$\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)=\frac{12}{25}$	$\frac{(2)\left(\frac{4}{5}\right)\left(\frac{3}{5}\right)(25)}{4}=6$
6	8	10	$\frac{1}{2}(6)(8)=24$	100	$\left(\frac{6}{10}\right)\left(\frac{8}{10}\right)=\frac{48}{100}$	$\frac{(2)\left(\frac{6}{10}\right)\left(\frac{8}{10}\right)(100)}{4}=24$
5	12	13	$\frac{1}{2}(5)(12)=30$	169	$\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)=\frac{60}{169}$	$\frac{(2)\left(\frac{5}{13}\right)\left(\frac{12}{13}\right)(169)}{4}=30$
1	1	$\sqrt{2}$	$\frac{1}{2}(1)(1)=\frac{1}{2}$	2	$\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)=\frac{1}{2}$	$\frac{(2)\left(\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{2}}\right)(2)}{4}=\frac{1}{2}$
1	$\sqrt{3}$	2	$\frac{1}{2}(1)(\sqrt{3})=\frac{\sqrt{3}}{2}$	4	$\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)=\frac{\sqrt{3}}{4}$	$\frac{(2)\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)(4)}{4}=\frac{\sqrt{3}}{2}$

Through Table No. (1), which appears by calculating the area of a right-angled triangle using Rule No. 1 and Rule 3 that the researcher reached in this study, and which we note that Rule No. 3 reached by the researcher gives the same result as the original rule No. 1, which confirms the validity of The rule reached by the researcher

Conclusions

We note through the rule reached by the researcher the following: He was able to calculate the area of a right-angled triangle in terms of the length of the hypotenuse and one of its acute angles. The student is presented with a new method for calculating the area of a right-angled triangle.

Recommendations

The researcher believes that there are still many scientific rules, especially in mathematics, that push us to search and investigate to find and use them, but we need patience, deliberation and careful consideration of mathematics and its beautiful forms and expressive symbols to present to humanity a lot of what is still hidden in our beautiful mathematical world.

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