



 Research Article

INFLUENCE OF SHOCK LOADING ON THE INFINITE PIECEWISE-HOMOGENEOUS TWO-LAYER PLATE

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ABSTRACT

This article examines the effect of normal load on an infinite piecewise homogeneous two-layer plate when the materials of the upper and lower layers of the plate are elastic. The transverse displacement of the points of the contact plane of a two-layer plate is determined, satisfying the approximate equation obtained in the work, replacing only the viscoelastic operators with elastic Lamé coefficients, respectively. For a rectangular infinite two-layer piecewise homogeneous plate under non-zero initial conditions, the frequencies of natural oscillations are calculated, and an analytical solution to this problem is constructed. The theoretical results obtained for solving dynamic problems of transverse vibration of piecewise homogeneous two-layer plates of constant thickness, taking into account the elastic properties of their material, make it possible to more accurately calculate the transverse displacement of the points of the contact plane of the plates under normal external loads.

KEYWORDS

Vibration equations, two-layer plate, displacement, elastic, viscoelastic, boundary conditions, initial conditions, operator, Lamé coefficients, differential equation.

INTRODUCTION

In real structures, the destruction of their elements is usually accompanied by impact loads.

In this work, a solution is constructed on the vibrations of an infinite two-layer plate under the

action of a normal load applied to the surface of a two-layer plate [5-12]. The problem is reduced to solving an approximate equation for the transverse displacement W of points of the

contact plane of a two-layer plate of constant thickness, obtained in [1] and [2].

$$Q_1 \left(\frac{\partial^4 W}{\partial t^4} \right) + Q_2 \left(\Delta \frac{\partial^2 W}{\partial t^4} \right) + Q_3 (\Delta^2 W) + Q_4 \left(\frac{\partial^6 W}{\partial t^6} \right) + Q_5 \left(\Delta \frac{\partial^4 W}{\partial t^4} \right) + Q_6 \left(\Delta^2 \frac{\partial^2 W}{\partial t^2} \right) + Q_7 (\Delta^3 W) = F(x, y, t) \quad (1)$$

Here the coefficients Q_j are determined by the formula obtained in [2-4].

Assuming the load $F(x, y, t)$ to be even in (x, y) , the transverse displacement W will be sought in the form of the Fourier integrals

$$W = \int_0^\infty \int_0^\infty W_0 \cos(kx) \cos(qy) dk dq \quad (2)$$

Substituting (2) into equations (1), for W_0 we obtain the ordinary differential equation

$$W_0^{VI} + A_1 W_0^{IV} + A_2 W_0^{II} + A_3 W_0 = F_0(k, q, t), \quad (3)$$

where the coefficients A_j and $F_0(k, q, t)$ are equal:

$$A_1 = \frac{Q_1' - \gamma^2 Q_5'}{Q_4'}; \quad A_2 = \frac{\gamma^2 (Q_2' - \gamma^2 Q_6')}{Q_4'}; \quad A_3 = \frac{\gamma^4 (Q_3' - \gamma^2 Q_7')}{Q_4'}$$

$$F_0(k, q, t) = \int_0^\infty \int_0^\infty F(x, y, t) \cos(kx) \cos(qy) dx dy,$$

and the coefficients Q_j' are determined by the formulas

$$Q_1' = P_2^2 (1 + hp)^2;$$

$$Q_2' = -2P_2^2 (2(P_2 D_0 + hD_1)(1 + hp) + (P_2 - 1)((1 + h) - (D_0 + hD_1 \rho)));$$

$$Q_3' = 4(P_2 - 1)(P_2 D_0 + h^2 D_1 + 2hP_2 D_0);$$

$$Q_4' = -\frac{1}{6} P_2^2 ((3h^2 \rho^2 + (1 + 4hp))(2 - D_0) + h^2 P_2 (3 + hp(hp + 4))(2 - D_1)); \quad (4)$$

$$Q_5' = -\frac{1}{6}P_2((2P_2(4D_0(1-D_0)+1)+(P_2-1)(4-D_0^2)) - \\ -P_2h^2\rho^2(2(4D_1^2-4D_1-1)-(P_2-1)D_1(2-D_1)) + \\ +6h^2(\rho(4(P_2D_0+D_1)+(P_2-1)(2P_2(1-D_0)-P_2D_1(2-D_0)+ \\ +D_1(1+D_0))))+P_2(1+\rho^2))+2h(2P_2\rho(2+4D_0-D_0^2)- \\ -h^2(2P_2-P_2D_1+5D_1-D_1^2))+(P_2-1)(4-3D_0)+ \\ +2D_1(4-D_0))+2P_2h\rho^2D_0(4-D_1));$$

$$Q_6' = \frac{1}{3}P_2(2D_0(3P_2-4D_0-1)+(P_2-1)(2+9D_0-3D_0^2)) + \\ +h_1^4P_2\rho(4D_1(1-2D_1)-4D_1+(P_2-1)D_1(3-D_1)) + \\ +3h^2(4P_2D_0(P_2(1-D_1)-D_1)-(P_2-1)(2(P_2-1)D_1(1-D_0)- \\ -P_2(2-D_0-4D_0D_1))+P_2\rho(4D_1(1+D_0+P_2D_0)- \\ -(P_2-1)(6D_0D_1(P_2-1)-6P_2D_0+D_1)))) + \\ +2hP_2(2(2D_1(1+2D_0)+(P_2-1)(1+2D_0-D_0^2)) + \\ +h_1^2(2(P_2-1)+D_1(P_2+3))-4P_2\rho D_0(1+h_0^2(2(P_2-1)(1-D_1)+P_2D_1+(1+D_1)))));$$

$$Q_7' = \frac{2}{3}(P_2D_0(4D_0-5(P_2-1)+h_1^4D_1(4D_1-(P_2-1))- \\ -3h^2(8P_2D_0D_1+(P_2-1)(3P_2D_0-(2P_2+1)D_0D_1-D_1(1-D_1))) - \\ -4hP_2D_0(2D_1+(P_2-1))+h_1^2(2(P_2-1)+(P_2+1)D_1)));$$

and γ is determined by the formula

$$\gamma^2 = h_0^2(k^2 + q),$$

and immeasurable parameters were introduced:

$$h = \frac{h_1}{h_0}; \rho = \frac{\rho_1}{\rho_0}; b = \frac{b_0}{b_1}; P_2 = \frac{\mu_0}{\mu_1}; D_0 = \frac{1}{2(1-v_0)}; D_1 = \frac{1}{2(1-v_1)}.$$

For ξ from equation (3) we obtain the frequency equation

$$\xi^6 + A_1\xi^4 + A_2\xi^2 + A_3 = 0 \quad (5)$$

frequency equation (5) has purely imaginary roots, i.e., frequencies of own fluctuations.

Then, the common decision of the homogeneous differential equation (4) is equal

$$W_{og} = C_1 \cos(\xi_1 t) + C_2 \sin(\xi_1 t) + C_3 \cos(\xi_2 t) + C_4 \sin(\xi_2 t) + C_5 \cos(\xi_3 t) + C_6 \sin(\xi_3 t). \quad (6)$$

Applying a method of a variation of any constants, for C'_j we will receive:

$$\begin{aligned} C'_1 &= \frac{1}{\xi_1 (\xi_1^2 - \xi_2^2) (\xi_1^2 - \xi_3^2)} F_0 \sin(\xi_1 t); \\ C'_2 &= -\frac{1}{\xi_1 (\xi_1^2 - \xi_2^2) (\xi_1^2 - \xi_3^2)} F_0 \cos(\xi_1 t); \\ C'_3 &= -\frac{1}{\xi_2 (\xi_1^2 - \xi_2^2) (\xi_1^2 - \xi_3^2)} F_0 \sin(\xi_2 t); \\ C'_4 &= \frac{1}{\xi_2 (\xi_1^2 - \xi_2^2) (\xi_1^2 - \xi_3^2)} F_0 \cos(\xi_2 t); \\ C'_5 &= \frac{1}{\xi_3 (\xi_2^2 - \xi_3^2) (\xi_1^2 - \xi_3^2)} F_0 \sin(\xi_3 t); \\ C'_6 &= -\frac{1}{\xi_3 (\xi_2^2 - \xi_3^2) (\xi_1^2 - \xi_3^2)} F_0 \cos(\xi_3 t). \end{aligned} \quad (7)$$

private decision of the differential equation (3) we will write down in a kind

$$\begin{aligned} W &= \frac{1}{(\xi_1^2 - \xi_2^2) (\xi_2^2 - \xi_3^2) (\xi_3^2 - \xi_1^2)} \left\{ \frac{\xi_2^2 - \xi_3^2}{\xi_1} \int_0^t F_0(k, q, \zeta) \sin[\xi_1(t - \zeta)] d\zeta + \right. \\ &+ \frac{\xi_3^2 - \xi_1^2}{\xi_2} \int_0^t F_0(k, q, \zeta) \sin[\xi_2(t - \zeta)] d\zeta + \\ &+ \left. \frac{\xi_1^2 - \xi_2^2}{\xi_3} \int_0^t F_0(k, q, \zeta) \sin[\xi_3(t - \zeta)] d\zeta \right\}. \end{aligned} \quad (8)$$

Satisfying with a zero initial condition, i.e.,

$$W_0 = \frac{\partial W_0}{\partial t} = \frac{\partial^2 W_0}{\partial t^2} = \dots = \frac{\partial^5 W_0}{\partial t^5} = 0, \quad (9)$$

We find that $C'_1 = C'_2 = \dots = C'_6 = 0$. then, the decision of a problem for displacement W looks like

$$\begin{aligned}
 W = & \int_0^\infty \int_0^\infty \frac{\cos(kx) \cos(qy)}{(\xi_1^2 - \xi_2^2)(\xi_2^2 - \xi_3^2)(\xi_3^2 - \xi_1^2)} \left\{ \frac{\xi_2^2 - \xi_3^2}{\xi_1} \times \right. \\
 & \times \int_0^t F_0(k, q, \zeta) \sin[\xi_1(t - \zeta)] d\zeta + \\
 & + \frac{\xi_3^2 - \xi_1^2}{\xi_2} \int_0^t F_0(k, q, \zeta) \sin[\xi_2(t - \zeta)] d\zeta + \\
 & \left. + \frac{\xi_1^2 - \xi_2^2}{\xi_3} \int_0^t F_0(k, q, \zeta) \sin[\xi_3(t - \zeta)] d\zeta \right\} dk dq
 \end{aligned} \tag{10}$$

Let, if

$$F(x, y, t) = \sigma_0 \delta(x) \delta(y) \delta(z),$$

Here – a σ_0 constant of dimension of pressure;

$\delta(\zeta)$ - delta - function of Diraka.

Then, problem decisions will register in a kind

$$\begin{aligned}
 W = & \sigma_0 \int_0^\infty \int_0^\infty \frac{\cos(kx) \cos(qy)}{(\xi_1^2 - \xi_2^2)(\xi_2^2 - \xi_3^2)(\xi_3^2 - \xi_1^2)} \left[\frac{\xi_2^2 - \xi_3^2}{\xi_1} \sin(\xi_1 t) + \right. \\
 & \left. + \frac{\xi_3^2 - \xi_1^2}{\xi_2} \sin(\xi_2 t) + \frac{\xi_1^2 - \xi_2^2}{\xi_3} \sin(\xi_3 t) \right] dk dq
 \end{aligned} \tag{11}$$

CONCLUSIONS

From the analytical decision of a problem on influence of normal loading on a surface of a two-layer plate follows that the deflection depends on geometrical and mechanical characteristics of a material of a plate, and also allows to describe precisely enough tensely - the deformed status of a plate in its any point eventually.

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