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Research Article

STOCHASTIC ASSET PRICING MODELS

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ABSTRACT

This article outlines asset pricing models. In these models, the price of an asset changes randomly over time. The initial models are very simple – price fluctuations are binomial. Based on these models, more complex ones are shown, which already have practical significance and are used in real financial calculations.

KEYWORDS

Floating interest rate, random variable, bank account, equation for stock price dynamics, Brownian motion, payoff function.

INTRODUCTION

The simplest binomial model

There is opinion among practitioners – financiers the prices follow certain rhythms, cycles, trends. Nowadays, with the development of computer technology and computer networks that connect the whole world into a single whole, price behavior can be seen on a computer screen in real

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time. The so-called technical analyses claims that certain parts of the price charts are repeated, and from the initial section of such a characteristic pattern, one can understand how the chart will go further. This is the possibility of predicting price behavior. In order to answer the question of whether price movements are predictable, many studies have been carried out. They brought an unexpected and paradoxical result: most likely, prices change completely randomly, approximately in the same way as the speeds of gas molecules change in chaotic Brownian motion. This question has not been finally resolved and, apparently, mill never be resolved, since again and again successful financiers will appear, confident that they can predict the future behavior of prices.

This article outlines 4 asset pricing models. In these models, the price of an asset changes randomly over time. The first two models are very simple – price fluctuations have only two values, which is why these models are called binomial. On the basis of these models, more complex ones are built, which already have practical significance and are used in real financial calculations. In this model, the s – price of an asset without any special restrictions, such as the price of a bond with redemption (at the time of maturity, the price is equal to the face value of the bond), for example, this is the price of a share. Let the unit of time be a day. Then the price of the asset by the end of the n-th day will be $S = S_0 + x_1 + \ldots + x_n$ where S_0 – is the price at the beginning of the observation, x_i $i = 1,2\ldots n$ - independent and equally distributed random variables that take the values $-1, +1$ with a probability of 0,5.

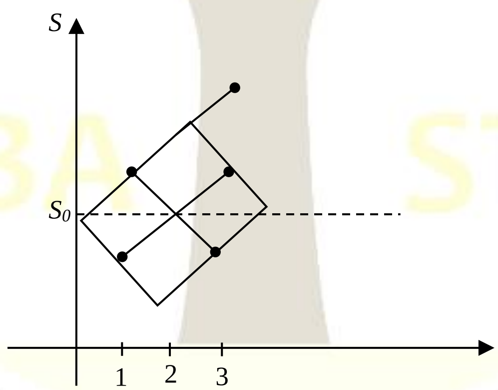

Figure – 1 shows the so-called binomial tree. Price behavior can be represented as a random movement along this tree from left to right. Let's find the mathematical expectation and variance of the random variable S_n. We have

$$M[S_n] = M[S_0] + \sum_{i=1}^{n} M[x_i] = S_0$$

since the mathematical expectation of each random x_i is 0. Further, due to the

fig – 1. Binomial tree. independence of the random variable and x_i, the variance of their sum is equal to the sum of their variances. But the variance of each random variable x_i is 1, hence $D[S_n]=n$.

Denote $x_1+\ldots+x_n$ by X_n. The probability that out of "n" random variable x_i k took the value +1, and the remaining $(n-k)$ took the value – 1, is equal to $C_n^k(0{,}5)^n$. The distribution series X_1, X_2, X_3 are shown in Fig – 2.

X_1	-1	1	X_2	-2	0	2	X_3	-3	-1	1	3
P	0,5	0,5	P	0,25	0,5	0,25	P	0,125	0,375	0,378	0,125

Fig – 2. Distribution series X_1, X_2, X_3

For $n>10$, one can already use the central theorem, which states that the sun of a large number of independent and identically distributed terms is approximately distributed according to the normal law.

$$P(\alpha<S_n-S_0<\beta)\approx\Phi\left(\beta/\sqrt{n}\right)-\Phi\left(\alpha/\sqrt{n}\right)$$

where Φ is the Laplace function. It follows that for $n>10$ $P\left(|S_n-S_0|<3\sqrt{n}\right)=0{,}9973$.

In particular, with $n=16$ we have $P(|S_n-S_0|<12)=0{,}9973$, i.e. in 16 days the price will change by no more than 12 units (it is assumed that S_0 significantly exceeds 12).

In this simplest model, prices cannot rise systematically, as, for example, the price of a zero-coupon bond rises as it nears redemption. It is also clear that the expected return on an asset is 0. Therefore, the risk-free rate must be equal to 0 (many observations show that the expected return on any risky asset cannot be less than the risk-free rate). All these considerations make this model suitable only for some explanatory illustrative calculations.

Binomial Cox-Ross-Rubinstein model

Suppose that we have two types of assets at our disposal. A bank account of value "B" with a constant interest rate "r", such that its value at the end of the n –th time period is equal to $B_n=(1+r)^n B_0$ and an asset of value S with a random rate of return f_i. Here the rates f_i - are independent and identically distributed random variables, taking two values - a, b, and $a<r<b$ with probabilities q and p, i.e. the interest rate is floating. In this case, the price of the asset at time n is equal t_0 $S_0\prod_{i=1}^{n}(1+f_i)$.

Let $b=\lambda-1,\ \alpha=1/\lambda-1$, where $\lambda>1$, we have

$$S_n=\begin{cases}\lambda S_{n-1}, & if \quad f_n=b\\ \lambda^{-1}S_{n-1}, & if \quad f_n=a\end{cases}$$

If we introduce *a* random variable $\varepsilon_m=\pm1$ with probability *q* and *p*, them

$$S_n=S_0\lambda^{\varepsilon_1+\ldots+\varepsilon_n}$$

Obviously, in this case, the price of the asset *S* wanders over the set $\{S_0\lambda^k,\ k=\overline{1,n}\}$. Let's find the mathematical expectation of the price at the *n* –th moment of time:

$$S_n=S_0\prod_{i=1}^{n}(1+f_i)$$

Since the random the value $(1+f_i)$, $i=1,\ldots,n$, independent, the mean their works is equal to the product of their mathematical expectations, so

$$M[S_n]=S_0\prod_{i=1}^{n}M(1+f_i)=S_0[1+aq+bp]^n \qquad (1)$$

Note that the securities market is called risk-neutral if investing in a bank account and in stocks gives, on average, the same result. In our case, this means that if $S_0=B_0$, then the equality $S_0(1+r)^n=S_0[1+aq+bp]$. From here we can find the probability *p*, corresponding to such *a* market:

$$a(1-p)+bp=r \quad \text{or} \quad p=\frac{r-a}{b-a}$$

General exponential binomial model

In the course of research on the behavior of prices, it was found that it is not the prices themselves that randomly wander, but their logarithms, i.e.

$$S_n=S_0 l^{H_n}$$

where $H_n=h_1+\ldots+h_n$ and random variables are $h_i(i=1,\ldots,n)$ - independent and approximately the same.

From this we can conclude from the central limit theorem that the values of H_n for $n>10$ are distributed approximately according to the normal law. The parameters of this law: the mathematical expectation and variance are completely determined by the mathematical expectations of the random variable h_i and their variances. Let us replace discrete time with continuous time. Then, in particular, it

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turns out that for any moment t and any $T > t$ natural logarithm of the price ratio $S(t+T)/s(t)$ is distributed according to the normal law.

When the natural logarithm of a random variable is distributed according to the normal law, then the distribution of the random variable itself is called lognormal. It can be proved that if $\ln(Y)$ is normally distributed with parameters α, G, then $M[Y] = e^{\alpha + G^2/2}$ and $D(Y) = e^{2a+G^2}\left(e^{G^2} - 1\right)$. So, in the general binomial model, the ratio of prices over any time interval is distributed lognormally.

Asset pricing models with continuous time

The dynamics of the bank account B_t with continuous accrual of interest at the rate has the form:

$$B_t = B_0 e^{\delta t}, \qquad t \geq 0$$

Calculating the differential from both parts, we get $dB_t = \delta B_0 e^{\delta t}$ or $dB_t = \delta B_t dt$

From here we get: $dB_t / B_t = \delta t$, i.e. the relative capital gain is proportional to time and interest rate.

Samuelson in 1965 introduced a similar equation for the dynamics of stock prices S_t:

$$\frac{dS_t}{S_t} = \mu d\,t + GW(t), \qquad (2)$$

where μ - is the growth rate or rate of return, G - is the coefficient of volatility or random variability. Randomness in price fluctuations is described by value $d\,w(t)$ - stochastic differential from the process of Brownian motion $w(t)$, which is determined by the following properties:

1) Process increments $w(t)$ on non – overlapping time intervals are independent of each other;
2) The increment $w(t) - w(s)$ at $t > s$ has zero mathematical expectation and variance equal to $t - s$, i.e.

$$M\,(w(t) - w(s)) = 0, \qquad D(w(t) - w(s)) = t - s$$

3) $w(0) = 0$

It follows from the definition that

$M(d\,w(t)) = M(w(t+dt) - w(t)) = 0$ and $D(d\,w(t)) = dt$

The solution of stochastic equation (2) has the form

$$S(t) = S_0 e^{\mu t} \cdot e^{Gw(t) + G^2 t/2} \qquad (3)$$

which is now commonly called geometric or economic Brownian motion.

It is easy to show that $M\left(e^{Gw(t) - G^2 t/2}\right) = 1$ and therefore the market described by this model will be risk neutral if $\mu = r$, because then $M(S(t) = B(t))$ with $S_0 = B_0$. We also note that the first stochastic model

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of stock price dynamics was proposed by Bachelier in 1900, according to which $S(t)=S_0+\mu t+Gw(t)$, i.e. this model is close to the simplest binomial model.

Option Pricing in other stochastic models

Let $f_n(S_0, S_1,\ldots,S_n)$ in the binomial cox –Ross – Rubinstein model be the payoff function describing the obligations of the option seller. Then the fair price of option C_n (i.e., the premium that the buyer must pay to the seller for the right to own this option) is found from the condition

$$C_n(1+r)^n = M_*[f_n(S_0, S_1,\ldots,S_n)], \qquad (4)$$

where M_* is the mathematical expectation to the neutral market, i.e. when

$$P = P_* = \frac{r-a}{b-a}$$

Equality (4) means that the seller of the option, having received the amount C_n from the sale and deposited it in a bank account, must, by time n, compensate on average his obligations to the buyer of the option, i.e.

$$C_n(1+r)^{-n} = M_*[f_n(S_0, S_1,\ldots,S_n)]$$

The right side of the last equality means the average costs of the option seller, discounted to the time of its sale $(n=0)$. In the case of the European call option, the payoff function has the form $f_n = \max[(S_n - S_H, 0)]$, where S_H is the contractual price, and then for C_n a more specific expression can be obtained

$$C_n = S_0 B(\alpha,n,P_*^1) - S_H r^{-n} B(\alpha,n,P_*),$$

where $P_*^1 = \frac{b}{r}P_*,\qquad \alpha = 1+\left[\log\frac{S_H}{S_0 a^n}/\log\frac{b}{a}\right],\qquad B(\alpha,n,P)=P(X_{n,p}\ge\alpha),$

$X_{n,p}$ is a binomial distributed random variable taking values $k=0,1,\ldots,n$ with probabilities $C_n^k p^k q^{n-k}$.

In the Samuelsson model, the rational value of a call option C_T with payoff function $f_T=(S_T-S_H)^+$ was obtained by Black and Scholes in 1973 and their famous formula is

$$C_t = S_0\Phi(d_1) - S_E e^{-\delta T}\Phi(d_2)^n, \qquad (5)$$

where

$$d_1 = \left[\log\frac{S_0}{S_E} + \left(r+\frac{G^2T}{2}\right)\right]/G\sqrt{T},\quad d_2 = d_1 - G\sqrt{T},$$

$\Phi(x)-$ is the distribution function of the standard normal distribution.

In the Black – Scholes formula, the value $\Phi(d_1)$ determines the sensitivity of the value of the option C to the value of the share S (when the stock price rises by 1 point (dollar, soum) the value of the option to buy the right rises by $\Phi(d_1)$ points). In addition, a portfolio of one stock and m call options will be risk – free if $m = 1/\Phi(d_1)$. The value $\Phi(d_2)$ is interpreted as the probability of the option being realized, i.e. the probability that the execution price S_E will exceed the share price at time T.

Example. Let the risk – free annual continuous interest rate $\delta = 0{,}1$, the initial cast of the share $S_0 = 100\$$, the maturity period 7 months (i.e. $T = 210/365$). Find the national value of the option for the right to buy at a price $S_E = 70\$$, if the volatility coefficient is $G = 0{,}8$.

Solution. Let's use the Black – Scholes formula. In our conditions

$d_1 = 0{,}986$; $d_2 = 0{,}379$; $\Phi(d_1) = 0{,}838$ $\Phi(d_2) = 0{,}648$. From formula (5) we get

$$C = 100 \cdot 0{,}838 - 70 \cdot e^{-0{,}1 \cdot 0{,}5753} \cdot 0{,}648 = 40{,}99\$.$$

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