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Research Article

EXACT THREE-DIMENSIONAL BOUNDARY VALUE PROBLEM FOR PIECEWISE HOMOGENEOUS PLATES

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ABSTRACT

In this work, in a general three-dimensional formulation, the problem of vibration of two-layer piecewise homogeneous viscoelastic plates of constant thickness is formulated. General equations of vibration are derived, expressions are given for displacements and stresses at the internal points of the plate through functions that describe the displacements and deformations of the points of the contact plane.

KEYWORDS

Three-dimensional, two-layer, plate, equation, vibrations, displacement, deformation.

INTRODUCTION

Plates are one of the main elements of many technical and building structures.

In many cases, the plates are non-uniform in thickness, in particular, piecewise homogeneous (two-layer, etc.) [1-4].

At present, there is practically no theory of vibration of piecewise homogeneous plates, and therefore the development of the theory and methods for calculating such plates is an urgent problem in structural mechanics [5-9].

Let us consider a piecewise-homogeneous viscoelastic plate of constant thickness, as a piecewise-homogeneous layer of the same geometry, with the thickness of the upper component being equal to h_0, and the thickness of the lower component h_1. The plate occupies the

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area - $\infty < (x,y) < \infty$; $-h_1 \le z \le h_0$, while the homogeneity interface coincides with the plane $z = 0$.

THE MAIN PART

The movement of the material of the constituent layers of the plate in Cartesian coordinates (x, y, z) is described by the equations of motion in stresses [10-16].

$$\frac{\partial \sigma_{xx}^{(k)}}{\partial x} + \frac{\partial \sigma_{xy}^{(k)}}{\partial y} + \frac{\partial \sigma_{xz}^{(k)}}{\partial z} = \rho_k \frac{\partial^2 u^{(k)}}{\partial t^2};$$

$$\frac{\partial \sigma_{xy}^{(k)}}{\partial x} + \frac{\partial \sigma_{yy}^{(k)}}{\partial y} + \frac{\partial \sigma_{yz}^{(k)}}{\partial z} = \rho_k \frac{\partial^2 \upsilon^{(k)}}{\partial t^2}; \quad (1)$$

$$\frac{\partial \sigma_{xz}^{(k)}}{\partial x} + \frac{\partial \sigma_{zy}^{(k)}}{\partial y} + \frac{\partial \sigma_{zz}^{(k)}}{\partial z} = \rho_k \frac{\partial^2 w^{(k)}}{\partial t^2};$$

Where $\sigma_{ij}{}^{(\kappa)}$ are the components of the stress tensor; $u^{(k)}$, $\upsilon^{(\kappa)}$, $w^{(\kappa)}$ are the components of the displacement vector.

In this case, stresses, displacements, and density in each of the layers will be denoted by the corresponding index “0” or “1”, i.e., k takes the values “0” and “1”.

The dependences of stress $\sigma_{ij}{}^{(k)}$ on deformations $\varepsilon_{ij}{}^{(k)}$ at points of the plate are described by linear operator equations, that is, we will assume that they are specified in the form of Boltzmann relations:

$$\sigma_{ij}^{(k)} = L_k(\varepsilon^{(k)}) + 2M_k(\varepsilon_{ij}^{(k)});$$

$$\sigma_{ij}^{(k)} = M_k(\varepsilon_{ij}^{(k)}); \ (i \ne j; \ \ i,j = x,y,z). \quad (2)$$

Where are the viscoelastic operators L_k and M_k – linear integral operators of the form

$$L_k(\zeta) = \lambda_k \left[\zeta(t) - \int_0^t f_1^{(k)}(t-\xi)\zeta(\xi)d\xi \right];$$

$$M_k(\zeta) = \mu_k \left[\zeta(t) - \int_0^t f_2^{(k)}(t-\xi)\zeta(\xi)d\xi \right]; \quad (3)$$

where $f_j^{(k)}(t)$ are the kernels of viscous operators, λ_k, μ_k – elastic constants or Lamé coefficients.

Let us introduce the potentials $\Phi^{(k)}$ and $\vec{\Psi}^{(k)}$ longitudinal and transverse waves according to the formula:

$$\vec{U}^{(k)} = grad\Phi^{(k)} + rot\vec{\Psi}^{(k)}$$

$$\vec{U}^{(k)} = \vec{U}^{(k)}(u^{(k)}, v^{(k)}, w^{(k)}). \quad (4)$$

Where $\vec{U}^{(k)}$ - vector of movement of plate points.

Then, instead of equations (1), we obtain integro-differential equations:

$$N_k(\Delta\Phi^{(k)}) = \rho_k \frac{\partial^2\Phi^{(k)}}{\partial t^2}; \quad M_k(\Delta\vec{\Psi}^{(k)}) = \rho_k \frac{\partial^2\vec{\Psi}^{(k)}}{\partial t^2}; \quad (5)$$

where the operator N_k is equal

$$N_k = L_k + 2M_k$$

Δ – three-dimensional Laplace operator

$$\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}.$$

By virtue of the Helmholtz theorem /4/, in the absence of internal sources, the vector potential $\vec{\psi}^{(k)}$ of transverse waves must satisfy the condition

$$div\vec{\Psi}^{(k)} = 0; \; \vec{\Psi}^{(k)} = \vec{\Psi}^{(k)}(\Psi_1^{(k)}, \Psi_2^{(k)}, \Psi_3^{(k)}). \quad (6)$$

Condition (6) for the vector components $\vec{\psi}^{(k)}$ takes the form

$$\frac{\partial\psi_1^{(k)}}{\partial x} + \frac{\partial\psi_2^{(k)}}{\partial y} + \frac{\partial\psi_3^{(k)}}{\partial z} = 0, \quad (7)$$

That is, we obtain a closing equation for determining the vector potential $\vec{\psi}^{(k)}$.

Equations (5) and condition (6) are sufficient to find general solutions for the scalar and vector potentials $\Phi^{(k)}$ and $\vec{\psi}^{(k)}$.

Vibrations of a viscoelastic piecewise homogeneous plate are caused by external forces applied to the surfaces of the plate. Therefore, the boundary conditions take the form:

at $z = h_0$ (on the upper surface of the plate)

$$\sigma_{zz}^{(0)} = f_{zz}^{(0)}(x,y,t); \quad \sigma_{xz}^{(0)} = f_{xz}^{(0)}(x,y,t); \; \sigma_{yz}^{(0)} = f_{yz}^{(0)}(x,y,t); \quad (8)$$

at $z = 0$ (contact plane)

$$\sigma_{zz}^{(0)} = \sigma_{zz}^{(1)}; \; \sigma_{xz}^{(0)} = \sigma_{xz}^{(1)}; \; \sigma_{yz}^{(0)} = \sigma_{yz}^{(1)};$$

$u^{(0)} = u^{(1)};\ \upsilon^{(0)} = \upsilon^{(1)};\ w^{(0)} = w^{(1)};$ (9)

at $z = -h_1$ (on the bottom surface of the record)

$\sigma_{zz}^{(1)} = f_z^{(1)}(x,y,t);\ \sigma_{xz}^{(1)} = f_{xz}^{(1)}(x,y,t);\ \sigma_{yz}^{(1)} = f_{yz}^{(1)}(x,y,t).$ (10)

The initial conditions of the problem are zero, that is

$$\frac{\partial \Phi^{(k)}}{\partial t} = \Phi^{(k)} = \frac{\partial \vec{\Psi}^{(k)}}{\partial t} = \vec{\Psi}^{(k)} = 0. \quad (11)$$

Displacements $u^{(k)}$, $\upsilon^{(k)}$ strains $\varepsilon_{ij}^{(k)}$ and stresses $\sigma_{ij}^{(k)}$ in Cartesian coordinates through the potentials $\Phi^{(k)}$ of $\vec{\psi}^{(k)}$ both longitudinal and transverse waves are determined by the following formulas /5/ .

For travel:

$$u_k = \frac{\partial \Phi^{(k)}}{\partial x} + \frac{\partial \Psi_3^{(k)}}{\partial y} - \frac{\partial \Psi_2^{(k)}}{\partial z};\ \upsilon_k = \frac{\partial \Phi^{(k)}}{\partial y} + \frac{\partial \Psi_1^{(k)}}{\partial z} - \frac{\partial \Psi_3^{(k)}}{\partial x};$$

$$w_k = \frac{\partial \Phi^{(k)}}{\partial z} + \frac{\partial \Psi_2^{(k)}}{\partial x} - \frac{\partial \Psi_1^{(k)}}{\partial y};\ (12)$$

For deformations:

$$\varepsilon_{xx}^{(k)} = \frac{\partial^2 \Phi^{(k)}}{\partial x^2} + \frac{\partial^2 \Psi_3^{(k)}}{\partial x \partial y} - \frac{\partial^2 \Psi_2^{(k)}}{\partial y \partial z};$$

$$\varepsilon_{yy}^{(k)} = \frac{\partial^2 \Phi^{(k)}}{\partial y^2} + \frac{\partial^2 \Psi_1^{(k)}}{\partial y \partial z} - \frac{\partial^2 \Psi_3^{(k)}}{\partial y \partial x};\ (13)$$

$$\varepsilon_{zz}^{(k)} = \frac{\partial^2 \Phi^{(k)}}{\partial z^2} + \frac{\partial^2 \Psi_2^{(k)}}{\partial x \partial z} - \frac{\partial^2 \Psi_1^{(k)}}{\partial y \partial z};$$

$$\varepsilon_{xy}^{(k)} = 2\frac{\partial^2 \Phi^{(k)}}{\partial x \partial y} + \frac{\partial^2 \Psi_1^{(k)}}{\partial x \partial z} - \frac{\partial^2 \Psi_2^{(k)}}{\partial y \partial z} + \frac{\partial^2 \Psi_3^{(k)}}{\partial y^2} - \frac{\partial^2 \Psi_3^{(k)}}{\partial x^2};\ (13)$$

$$\varepsilon_{yz}^{(k)} = 2\frac{\partial^2 \Phi^{(k)}}{\partial y \partial z} + \frac{\partial^2 \Psi_2^{(k)}}{\partial x \partial y} - \frac{\partial^2 \Psi_2^{(k)}}{\partial x \partial z} + \frac{\partial^2 \Psi_1^{(k)}}{\partial z^2} - \frac{\partial^2 \Psi_1^{(k)}}{\partial y^2};$$

$$\varepsilon_{xz}^{(k)} = 2\frac{\partial^2 \Phi^{(k)}}{\partial x \partial z} + \frac{\partial^2 \Psi_3^{(k)}}{\partial y \partial z} - \frac{\partial^2 \Psi_2^{(k)}}{\partial z^2} + \frac{\partial^2 \Psi_2^{(k)}}{\partial x^2} - \frac{\partial^2 \Psi_1^{(k)}}{\partial x \partial y};$$

For voltages:

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$$\sigma_{xx}^{(k)} = L_k(\varDelta\varPhi^{(k)}) + 2M_k(\frac{\partial^2\varPhi^{(k)}}{\partial x^2} + \frac{\partial^2\varPsi_3^{(k)}}{\partial x\partial y} - \frac{\partial^2\varPsi_2^{(k)}}{\partial x\partial z});$$

$$\sigma_{yy}^{(k)} = L_k(\varDelta\varPhi^{(k)}) + 2M_k(\frac{\partial^2\varPhi^{(k)}}{\partial y^2} + \frac{\partial^2\varPsi_1^{(k)}}{\partial y\partial z} - \frac{\partial^2\varPsi_3^{(k)}}{\partial x\partial y}); \quad (14)$$

$$\sigma_{zz}^{(k)} = L_k(\varDelta\varPhi^{(k)}) + 2M_k(\frac{\partial^2\varPhi^{(k)}}{\partial z^2} + \frac{\partial^2\varPsi_2^{(k)}}{\partial x\partial z} - \frac{\partial^2\varPsi_1^{(k)}}{\partial y\partial z});$$

$$\sigma_{xy}^{(k)} = M_k(2\frac{\partial^2\varPhi^{(k)}}{\partial x\partial y} - \frac{\partial^2\varPsi_3^{(k)}}{\partial x^2} + \frac{\partial^2\varPsi_3^{(k)}}{\partial y^2} + \frac{\partial^2\varPsi_1^{(k)}}{\partial x\partial z} - \frac{\partial^2\varPsi_2^{(k)}}{\partial y\partial z});$$

$$\sigma_{yz}^{(k)} = M_k(2\frac{\partial^2\varPhi^{(k)}}{\partial y\partial z} - \frac{\partial^2\varPsi_1^{(k)}}{\partial y^2} + \frac{\partial^2\varPsi_1^{(k)}}{\partial z^2} + \frac{\partial^2\varPsi_2^{(k)}}{\partial x\partial y} - \frac{\partial^2\varPsi_3^{(k)}}{\partial x\partial z});$$

$$\sigma_{xz}^{(k)} = M_k(2\frac{\partial^2\varPhi^{(k)}}{\partial x\partial y} + \frac{\partial^2\varPsi_2^{(k)}}{\partial x^2} - \frac{\partial^2\varPsi_2^{(k)}}{\partial z^2} - \frac{\partial^2\varPsi_1^{(k)}}{\partial x\partial y} + \frac{\partial^2\varPsi_3^{(k)}}{\partial y\partial z});$$

Thus, the exact three-dimensional problem of vibration of a viscoelastic piecewise homogeneous plate of constant thickness is reduced to solving the vibration equations (5) in potentials $\varPhi^{(k)}$ and $\vec{\varPsi}^{(k)}$ under boundary conditions (8), (9), (10) and zero initial conditions (11).

Conclusions

1. The study of vibrations of piecewise homogeneous plates in an exact three-dimensional formulation allows, without involving any hypotheses, to derive the general and approximate equations of vibration of such plates based on them.

2. For piecewise homogeneous plates there is neither a purely transverse nor a purely longitudinal vibration; as shown, this is an equation of the sixth order in derivatives, which for a homogeneous plate of constant thickness becomes the product of two integro-differential operators describing longitudinal and transverse vibrations.

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