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## Research Article

# METHODS FOR SOLVING PROBLEMS BASED ON THE LAWS OF CONSERVATION OF ENERGY AND MOMENTUM

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## ABSTRACT

This article is devoted to one such theory – the theory of collision of bodies. The impact phenomenon is well described by a simple mathematical model. This article also discusses the issue of joint use of two conservation laws – energy and momentum – to solve various physical problems.

## KEYWORDS

Energy, impulse, elastic, inelastic, theory of collision of bodies, absolutely elastic impact, total impulse, total kinetic energy.

## INTRODUCTION

Some physical theories are built on the model adopted in mathematics – form a small number of physical statements that play the same role as axioms in mathematical theories, various

consequences are logically strictly deduced. This article is devoted to one such theory – the theory of collision of bodies. The impact phenomenon is well described by a simple mathematical model.

This article discusses the issue of joint use of two conservation laws – energy and momentum – to solve various physical problems. During exams in physics, a situation often arises when students, knowing well each conservation law separately, experience “psychological” difficulties when it is necessary to combine these laws together within the framework of one problem. Moreover, more often than not, the simpler, in our opinion, law of conservation of momentum escapes attention. Having written down the corresponding equation for energy, the student no longer remembers the impulse – and gets into trouble. Let’s consider

several specific examples of how the joint efforts of energy and impulse lead to the desired result.

**Example 1.** Two balls, made of the same material and having masses  $m_1$  and  $m_2$ , move towards each other with speeds  $V_1$  and  $V_2$ . Now much will the temperature of the balls increase after a frontal absolutely inelastic impact if the specific heat capacity of the material of the balls is  $C$ ? The initial temperatures of the balls were the same.

**Solution.** The change in the temperature of the balls is determined by the increase in the their internal energy:

$$\Delta E = c(m_1 + m_2)\Delta t \quad (1)$$

Many student mistakenly believe that as a result of an impact, all the initial kinetic energy of the system  $\frac{m_1 V_1^2}{2} + \frac{m_2 V_2^2}{2}$  is converted into integral energy. At the same time, they forget that the balls cannot stop after impact, since this would contradict the law of conservation of momentum – the initial momentum of the system  $m_1 \vec{V}_1 + m_2 \vec{V}_2$ , generally speaking, is not equal to zero. This means that when calculating energy, it is necessary to take into account the kinetic energy of the balls in the final state.

Let us denote the speed of the balls stuck together after an absolutely inelastic impact by  $v$  and write the laws of conservation of energy and momentum for the direction of motion of the first ball:

$$\frac{m_1 V_1^2}{2} + \frac{m_2 V_2^2}{2} = \frac{(m_1 + m_2) v^2}{2} + \Delta E \quad (2)$$

$$m_1 V_1 - m_2 V_2 = (m_1 + m_2) v \quad (3)$$

Solving the three equations obtained together, we find the desired temperature increase:

$$\Delta t = \frac{m_1 m_2 (V_1 + V_2)^2}{2c(m_1 + m_2)^2} \quad (4)$$

**Example 2.** Two cars, the masses of which are  $M_1$  and  $M_2$ , are moving towards each other with speed  $V_1$  and  $V_2$ . During a collision, four identical buffer springs move apart. Find the maximum deformation of each spring if its spring constant is  $k$ .

**Solution.** In this problem, unlike the previous one, you can use the law of conservation of mechanical energy (assuming that friction is low and the springs are ideal), equating the initial energy of the cars to the energy of the system at the moment when the deformation of the springs “ $x$ ” is maximum. In this case, the desired value “ $x$ ” will be included in the potential energy of elastic deformation of the springs:

$$E_p = 4 \frac{kx^2}{2} \quad (5)$$

However, in addition to this energy, it is also necessary to take into account the kinetic energy of the cars.

The fact that at maximum approach the cars do not stop (which, unfortunately, many students forget) follows, as in the previous problem from the law of conservation of momentum. The only peculiarity of this moment is that at maximum deformation of the springs from the speed of the cars are the same:  $V'_1 = V'_2 = V$ . Therefore, the laws of conservation of energy and momentum are as follows:

$$\frac{M_1 V_1^2}{2} + \frac{M_2 V_2^2}{2} = \frac{(M_1 + M_2) V^2}{2} + 4 \frac{kx^2}{2} \quad (6)$$

$$M_1 V_1 - M_2 V_2 = (M_1 + M_2) V \quad (7)$$

From here we get

$$X = \frac{1}{2} \sqrt{\frac{M_1 \cdot M_2}{K(M_1 + M_2)}} \cdot (V_1 + V_2) \quad (8)$$

**Example 3.** A block of mass  $M$ , hanging on parallel threads of length  $l$ , is hit by a horizontally flying bullet of mass  $m$  and gets stuck in it (Fig.1). As a result of the impact, each thread is deflected by angle  $\alpha$ . Find the initial speed of the bullet  $V$ . the threads are considered ideal (weightless and inextensible).

**Solution.** As can be seen from the figure, the angle of deflection of the threads  $\alpha$  is related to the height  $h$  to which the block rises:

$$h = l(1 - \cos \alpha)$$

and the height  $h$  can be related to the potential energy of the block and bullet in the final state:

$$E_p = (M + m)gh \quad (9)$$



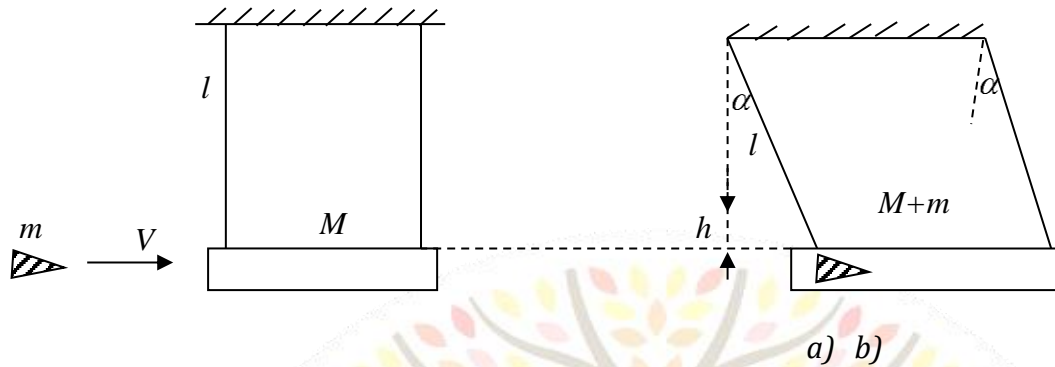


Fig. 1.

The question arises: is the law of conservation of mechanical energy satisfied in this situation? In other words, is the energy of the system in the final state equal to its initial energy, i.e. kinetic energy of the bullet  $\frac{mv^2}{2}$ ? The answer, of course, is negative. After all, we know that during an inelastic impact, part of the mechanical energy transforms into internal energy.

Now to be? Let's consider another, intermediate state of the system – immediately after the end of the impact, when the bullet is already stuck in the bar, but the threads are still vertical. The energy of the system this state is simply the kinetic energy of the block with the bullet:

$$E_k = \frac{(m+M)V'^2}{2}, \text{ where } V' - \text{ is their total}$$

speed. After the inelastic impact has already ended, no more energy will be lost, and we can write  $E_k = E_p$ , or

$$\frac{(m+M)V'^2}{2} = (M+m)gh$$

The speed  $V'$  can be related to the initial speed of the bullet using the law of conservation of momentum

$$mV = (m+M)V'$$

From the last two equations, taking into account the expression for  $h$ , we have

$$V = 2 \left( 1 + \frac{M}{m} \right) \sqrt{gl} \sin \frac{\alpha}{2}$$

Note that in this problem the laws of conservation of momentum and energy do not work simultaneously, but as if in turn. It turns out that this is not so easy to understand, and many students solve problems of this type using only the law of conservation of energy, obtaining, of course, incorrect results.

**Example 4.** On a block of length  $l$  and mass  $M$ , located on a smooth horizontal surface, lies a small body of mass  $m$  (fig.2). The coefficient of friction between the body and the block  $\mu$ . At what speed  $V$  must the system move so that after an elastic impact of the block on the wall the body falls from the block?

**Solution.** The impact of the block against the wall will cause its speed to abruptly change to the opposite. The speed of the body will not have time

to change during the impact, and it will begin to slide along the impact, and it will begin to slide along the block. Let us find the distance  $x$  the body will move relative to the block before the end of

sliding. It is clear that the condition  $x > l$  will be the condition for the body to fall from the block. The work done by the distance  $x$

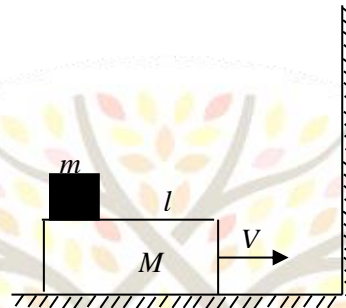


Fig. 2.

$$A = -\mu mgx,$$

which, in turn, is equal to the change in the kinetic energy of the system:

$$A = \frac{(m+M)V'^2}{2} - \left( \frac{mV^2}{2} + \frac{MV^2}{2} \right)$$

here  $V'$  - is the speed of the block with the body at the moment when the body stops relative to the block. This speed can be found from the law of conservation of momentum.

$$MV - mV = (M+m)V'$$

Solving all three equations together, we get

$$x = \frac{2MV^2}{\mu g(M+m)}.$$

The condition  $x > l$  allows you to find the required speed:

$$V > \sqrt{\frac{1}{2} \mu gl \left( 1 + \frac{m}{M} \right)}$$

**Example 4.** At the left edge of the tree with length  $L = 0,2m$  and mass  $M = 1kg$  lies a cube with mass  $m = 0,3 kg$  (Fig.3). the cube is given a push to the horizontal speed  $V_0 = 1m/s$  to the right. Assuming that the cart is stationary at the initial moment, determine at what distance from the left edge of the cart the cube will be after its sliding relative to the cart stops. The coefficient of friction between the cube and the walls is assumed to be absolutely elastic. The cart rides on the table without friction.

**Solution.** The easiest way to solve this problem is from energy considerations. According to the law of conservation of energy, the decrease in the kinetic energy of the system is equal to the amount of heat released, which, in turn, is equal to the work of the sliding friction force on the braking distance  $l$ :

$$\Delta E_x = \frac{(M + m)u^2}{2} - \frac{mV_0^2}{2} = Q = F_{fr} \cdot l = -\mu mgl.$$

The speed of the system  $U$  after the cessation of slipping is easy to find from the law of conservation of momentum

$$mV_0 = (M + m)U$$

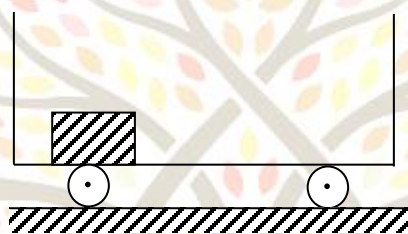


Fig. 3.

After simple transformations we get

$$l = \frac{V_0^2}{2\mu g \left(1 + \frac{m}{M}\right)} \approx 0,38$$

This means that the cube will stop at a distance

$$x = L - (l - L) = 0,02 \text{ m}$$

from the left edge of the cart.

## CONCLUSION

Galileo also carried out a series of experiments to clarify the laws of the collision of bodies. These experiments, however, did not lead him to definite conclusions. A contemporary of Galileo, the Prague professor Marzi, published in his work some of the results of his research into the phenomenon of impact. In particular, he knew that a body, having elastically hit an identical body at rest, loses its speed, imparting it to this body. The first detailed study of the laws of

impact was undertaken in 1668 at the suggestion of the Royal Society of London. Three outstanding mechanics and mathematicians Wallis, Rehn and Huygens presented their works in which they outlined the laws of motion of colliding bodies. John Wallis limited himself, without specifying this, to considering a completely inelastic impact. He proceeded from the hypothesis of conservation of the total momentum of colliding bodies. Christopher wren outlined the rules for calculating elastic impact. Wren, like Wallies, did not provide any theoretical considerations, but to test his rules he performed a number of simple



and convincing experiments. Newton referred to these experiments in his famous “Mathematical Principles of Natural Philosophy” (1687). Christian Huygens’s competition memoir was the most complete study of impact theory. It outlined the derivation of the relations of the impact theory, based on Galileo’s principle of relativity. The Royal Society of London published only the memoirs of Wallis and Renault. In Huygens memoir on the motion of bodies under the influence of an impact, the law of conservation of momentum is derived from Galileo’s principle of relativity. From a mathematical point of view, Huygens’s reasoning may not be considered completely rigorous.

## REFERENCES

1. Трофимова Т.И. Курс физики: Учебное пособие для вузов. 7-е изд. стер М.: Высш. шк., 2002, -542 ст.
2. Савельев И.В. Курс общей физики. Учебное пособие для вузов. СПб.: СпецЛит, 2002. 336 с.
3. Волькенштейн В.С. Сборник задач по общему курсу физики. Изд. Доп. и перераб.
4. Чертов А.Г., Воробьев А.А. Задачник по физике: Учеб. пособие для вузов. – 7-е изд., перераб. и доп. –М.: Издательство физико-математической литературы, 2003. – 640 с.