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Research Article

SOME METHODS OF CALCULATING N-ORDER DETERMINANTS AND WAYS TO SOLVE EXAMPLES RELATED TO THEM

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ABSTRACT

Certain methods used in this paper to calculate numerical determinants are computationally intensive. For certain forms of literal and numerical determinants, some methods of their calculation have been developed.

KEYWORDS

Determinant, triangle method, recurrent relations, major minor, diagonal view, determinant order.

INTRODUCTION

The main idea of the method of bringing the determinant to the form of a triangle is that all elements on one side of the diagonal are reduced to zero by performing elementary substitutions. If the elements lying on one side of the main diagonal are equal to zero, then such a determinant is equal to the product of all elements on the main diagonal. If all the elements lying on one side of the auxiliary diagonal of the determinant are equal to zero, then such a determinant is $(-1)^{\frac{n(n-1)}{2}}$ equal to the product of

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all the elements of the diagonal taken with the sign.

Example 1. Calculate the nth-order determinant.

$$d = \begin{vmatrix} a & a\ldots & a & a+x \\ a & a\ldots & a+x & a \\ \ldots & \ldots & \ldots & \ldots \\ a+x & a\ldots & a & a \end{vmatrix}$$

Solving. We add all previous columns to the last column:

$$d = \begin{vmatrix} a & a\ldots & a & na+x \\ a & a\ldots & a+x & na+x \\ \ldots & \ldots & \ldots & \ldots \\ a+x & a\ldots & a & na+x \end{vmatrix}$$

The common multiplier in the last column under the determinant sign is -

we subtract na + x. We subtract the last column multiplied by a from each of the previous columns. As a result, we arrive at the determinant in the form of a triangle, with all elements above the auxiliary diagonal consisting of zeros:

$$d = (na + x)\begin{vmatrix} 0 & 0 & \ldots & 0 & 1 \\ 0 & 0 & \ldots & x & 1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 0 & x & \ldots & 0 & 1 \\ x & 0 & \ldots & 0 & 1 \end{vmatrix}$$

So, $d = (-1)^{\frac{n(n-1)}{2}}(x+na)x^{n-1}$

The main idea of the method of separation of linear multipliers is to treat the n-order determinant as an m-order polynomial of one or more variables. One can find m mutually radical linear multipliers that divide the determinant directly or by performing certain substitutions. In that case, the determinant constant multiplier S is equal to the product of these linear multipliers. The constant number S is found as a result of comparing the term of the determinant and the term in the product of linear multipliers, respectively.

Example 2. Calculate the nth-order determinant.

$$d = \begin{vmatrix} 1 & 2 & 3 & \dots & n \\ 1 & x+a & 3 & \dots & n \\ 1 & 2 & x+a & \dots & n \\ \dots & \dots & \dots & \dots & \dots \\ 1 & 2 & 3 & \dots & x+a \end{vmatrix}$$

Solving. The product of elements on the diagonal of the determinant keeps x at the largest - (n – 1) level. So, this determinant is a polynomial of (n – 1) degree x. At x = 2 - a, x = 3 - a, ..., x = n - a, the 1st and 2nd, 1st and 3rd,. , 1st and nth lines of this determinant are the same will be the same, and as a result, the determinant will be zero. Thus, d determinant is divided by x + a - 2, x + a - 3, ..., x + a - n, and therefore,

d = c (x + a -2)(x + a -3)...(x + a – n) (*)

To find the number c, we compare the term xn-1 formed by multiplying the elements of the main diagonal with the term c xn-1 on the right side of (*). Given that these terms are equal, s = 1 and as a result

We form d = (x + a – 2) (x + a – 3)...(x + a – n).

In the method of recurrent relations, the given determinant is reduced to one or more determinants of the same order of small order. For this, the determinant is spread over a row or column. In some cases, the determinant is made convenient by making certain substitutions and then spreading it over rows or columns. An equality that expresses a determinant through one or more lower-order determinants in the same form is called recurrent or return equality. Using the method of mathematical induction, the general expression of the given determinant is derived from the recurrent equation.

This method can also be used in the following modified form: in the recurrent equation expressed by n-order determinants, the expression when replacing n in this recurrent equation with (n – 1) is given; similarly (n – 2)-order expression, etc. will be posted. As a result, the general view of the n-order determinant is formed. The correctness of this expression is checked using the method of mathematical induction.

Example 3. Calculate the nth-order determinant.

$$d_n = \begin{vmatrix} 7 & 4 & 0 & 0 & \dots & 0 & 0 \\ 3 & 7 & 4 & 0 & \dots & 0 & 0 \\ 0 & 3 & 7 & 4 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & 3 & 7 \end{vmatrix}$$

Solving. Spread along the first line, $d_n = 7d_{n-1} - 12d_{n-2}$ we generate . This is consistent with the recurrence relation $x^2 - 7x + 12 = 0$ quadratic equation $\alpha = 3, \ \beta = 4 \ (\alpha \neq \beta)$ has roots. So, $d_n = c_1 3^n + c_2 4^n$. We find the coefficients c1 and c2 from the formulas

$$c_1 = \frac{d_2 - \beta d_1}{\alpha(\alpha - \beta)}, \ c_2 = -\frac{d_2 - \alpha d_1}{\beta(\alpha - \beta)}. \ \ d_2 = \begin{vmatrix} 7 & 4 \\ 3 & 7 \end{vmatrix} = 37, \ \ d_1 = 7,, \ .$$

since c1=-3, c2=4. So, it will be $d_n = 4^{n+1} - 3^{n+1}$.

The method of expanding the determinant into the sum of determinants is sometimes easily calculated by expressing the n-order determinant in the form of the sum of two or more determinants.

Example 4. Calculate the nth-order determinant.

$$d = \begin{vmatrix} a & b & 0 & 0 & \dots & 0 & 0 \\ 0 & a & b & 0 & \dots & 0 & 0 \\ 0 & 0 & a & b & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & a & b \\ 0 & 0 & 0 & 0 & \dots & 0 & a \end{vmatrix}$$

Solving. Spread the determinant on the first column:

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$$d = a\begin{vmatrix} a & b & 0 & \dots & 0 \\ 0 & a & b & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & a \end{vmatrix} + (-1)^{n-1} b \begin{vmatrix} b & 0 & 0 & \dots & 0 \\ a & b & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & b \end{vmatrix} =$$

$$= a \cdot a^{n-1} + (-1)^{n+1} b \cdot b^{n-1} = a^n + (-1)^{n+1} b^n.$$

Both determinants have a triangular form.

The method of changing the elements of the determinant - in this method, by changing all the elements of the determinant to one number, it becomes convenient to calculate the algebraic complement of all elements. This method is based on the following property: if we add exactly one number x to all elements of the determinant d, then the number x of the determinant is d.

$$d = \begin{vmatrix} a_{11} & \dots & a_{1n} \\ \dots & \dots & \dots \\ a_{n1} & \dots & a_{nn} \end{vmatrix}, \quad d' = \begin{vmatrix} a_{11}+x & \dots & a_{1n}+x \\ \dots & \dots & \dots \\ a_{n1}+x & \dots & a_{nn}+x \end{vmatrix}$$

let it be into two determinants with respect to line 1, and each of them into two determinants with respect to line 2, etc. we write Determinants with more than one row of all elements equal to x are equal to zero, and determinants with one row of elements equal to x are spread over this row.

Then we form the equality that needs to be proved $d' = d + x\sum_{i,j=1}^{n} A_{ij}$. Thus, the calculation of the d' determinant is reduced to the calculation of the d determinant and the sum of its algebraic complements.

Calculating the nth-order determinant to the Vandermond determinant is the Vandermond determinant, is called the determinant in the form of

$$V_n = \begin{vmatrix} 1 & x_1 & x_1^2 & x_1^3 & \dots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & x_2^3 & \dots & x_2^{n-1} \\ \dots & \dots & \dots & \dots & \dots & \dots \\ 1 & x_n & x_n^2 & x_n^3 & \dots & x_n^{n-1} \end{vmatrix}$$

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It is calculated using the following formula:

$$V_n = (x_2 - x_1)(x_3 - x_1)\ldots(x_n - x_1)(x_3 - x_2)(x_4 - x_2)\ldots(x_n - x_2)\ldots(x_n - x_{n-1}) =$$
$$= \prod_{n \geq i \geq k \geq 1} (x_i - x_k).$$

Some determinants can be calculated by bringing them to the Vandermonde determinant.

Example 5. Calculate the determinant by multiplying by the Vandermond determinant.

$$d = \begin{vmatrix} \alpha_1^n & \alpha_1^{n-1} & \beta_1 & \ldots & \beta_1^n \\ \alpha_2^n & \alpha_2^{n-1} & \beta_1 & \ldots & \beta_2^n \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ \alpha_{n+1}^n & \alpha_{n+1}^{n-1} & \beta_{n+1} & \ldots & \beta_{n+1}^n \end{vmatrix}$$

Solving. Under the determinant sign $\alpha_1^n, \alpha_2^n, \ldots, \alpha_{n+1}^n$, we subtract the multipliers from the first, ..., (n+1) lines, respectively. As a result, we form the Vandermond determinant:

$$d = \alpha_1^n \alpha_2^n \ldots \alpha_{n+1}^n \begin{vmatrix} 1 & \frac{\beta_1}{\alpha_1} & \ldots & \left(\frac{\beta_1}{\alpha_1}\right)^n \\ 1 & \frac{\beta_2}{\alpha_2} & \ldots & \left(\frac{\beta_2}{\alpha_2}\right)^n \\ \ldots & \ldots & \ldots & \ldots \\ 1 & \frac{\beta_{n+1}}{\alpha_{n+1}} & \ldots & \left(\frac{\beta_{n+1}}{\alpha_{n+1}}\right)^n \end{vmatrix} = \alpha_1^n \alpha_2^n \ldots \alpha_{n+1}^n \cdot \prod_{i>j} \left[\left(\frac{\beta_i}{\alpha_i}\right) - \left(\frac{\beta_j}{\alpha_j}\right) \right] =$$

$$= \alpha_1^n \alpha_2^n \ldots \alpha_{n+1}^n \prod_{i>j} \frac{\alpha_j \beta_i - \alpha_i \beta_j}{\alpha_i \alpha_j} = \prod_{i>j} (\alpha_j \beta_i - \alpha_i \beta_j).$$

Conclusion

Currently, we know that in many areas, as mathematics enters, the issue of calculating it in the most optimal way is seen. The methods presented in this article deal with the issue of calculating the determinants given in a complex form, simplifying them. As an example, we can say that when calculating determinants, it is much easier to calculate them in a convenient way, except for the principals. In addition, modern professions are also used, by including the algorithm of these methods in the program, it will be possible to easily calculate many complex-looking determinants.

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