International Journal of Advance Scientific Research
(ISSN – 2750-1396)
VOLUME 04 ISSUE 06 Pages: 57-65
SJIF IMPACT FACTOR (2022: 5.636) (2023: 6.741) (2024: 7.874)
OCLC – 1368736135

Journal Website: http://sciencebring.com/index.php/ijasr

Research Article

METHODS FOR SOLVING UN CONDITIONAL AND CONDITIONAL EXTREMUM PROBLEMS

Submission Date: June 20, 2024, **Accepted Date:** June 25, 2024,
Published Date: June 30, 2024
Crossref doi: https://doi.org/10.37547/ijasr-04-06-11

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ABSTRACT

The article discusses conditional programming problems. Such problems can in principle, be solved using classical methods. However, along this path there are computational difficulties that make it necessary to search for other solution methods. Therefore, in this article we proposed particular methods for solving nonlinear programming problems.

KEYWORDS

Unconditional and conditional problems, extremum problems, greatest and least values of the product.

INTRODUCTION

If in a problem for an extremum

$$f(x_1, x_2, \dots x_n) \to max \qquad (1)$$

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there are no restrictions on variables, then such a problem is called an unconditional problem for an extremum. The following problem for the extremum.

$$f(x) \to min \qquad (2)$$

$$g_i(x) \geq 0; i = 1,2,\dots,m, x \in R^n$$

is called the conditional minimum problem of nonlinear programming.

Example - 1. Find all pairs (x, y) of positive numbers at which the smallest value of the function is achieved

$$f(x,y) = \frac{x^4}{y^4} + \frac{y^4}{x^4} - \frac{x^2}{y^2} - \frac{x^2}{y^2} + \frac{x}{y} + \frac{y}{x}$$

Solution. Since there are relations

$$f(x,y) - 2 = \left(\frac{x^2}{y^2} - 1\right)^2 + \left(\frac{y^2}{x^2} - 1\right)^2 + \left(\frac{x}{y} - \frac{y}{x}\right)^2 + \left(\frac{x}{y} - 2 + \frac{y}{x}\right) \geq \frac{(x-y)^2}{xy} \geq 0$$

and equality $f(x,y) = 2$ is achieved if and only if $x = y$.

Example - 2. If for positive numbers a, b, c and $abc = 1$, then find the minimum value expression those

$$\frac{1}{a^3(b+c)} + \frac{1}{b^3(c+a)} + \frac{1}{c^3(a+b)} \to min$$

Solution. Convenient to move to new variables $x = \frac{1}{a}, y = \frac{1}{b}, z = \frac{1}{c}$, also positive and related by condition $xyz = 1$. This expression is equivalent to the following:

$$S = \frac{x^2}{y+z} + \frac{y^2}{z+x} + \frac{z^2}{x+y} \to min$$

Applying the Cauchy-Bunyakovsky inequality to vectors

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$\bar{u} = \frac{x}{\sqrt{y+z}} + \frac{y}{\sqrt{z+x}} + \frac{z}{\sqrt{x+y}}$ and equality $\bar{v} = \left(\sqrt{y+z}, \sqrt{z+x}, \sqrt{x+y}\right)$ we obtain

$$\bar{u}\bar{v} \leq |\bar{u}||\bar{v}|$$

$(x+y+z)^2 \leq 2S(x+y+z)$, those $S \geq \frac{x+y+z}{2}$

Using the inequality between the geometric mean of three positive numbers we get:

$$S \geq \frac{1}{2}(x+y+z) \geq \frac{3}{2}\sqrt[3]{xyz} = \frac{3}{2}$$

Example - 3. For a given number $n \geq 2$, find the largest and smallest values of

$$x_1, x_2, \dots, x_n$$

provided that $x_i \geq \frac{1}{n}$ $(i = 1,2, \dots, n)$ and $x_1^2 + x_2^2 + \dots + x_n^2 = 1$

Solution. 1) Let's find the smallest value of the product $x_1, x_2, \dots, x_n$. Let an arbitrary set $(x_1, x_2, \dots, x_n)$ satisfy the conditional problem. Consider a new set $(y_1, y_2, \dots, y_n)$

where $y_1 = x_1, y_2 = x_2, \dots, y_{n-2} = x_{n-2}$

$$y_{n-1} = \sqrt{x_{n-1}^2 + x_n^2 - \frac{1}{n^2}}, \qquad y_n = \frac{1}{n}$$

This set satisfies the relations

$$y_i \geq \frac{1}{n}, (i = 1,2, \dots, n), \sum_{i=1}^{n} (y_i)^2 = 1$$

Let's prove

$$x_1 x_2 \dots x_n \geq y_1 y_2 \dots y_n$$

Indeed, we have

$$x_{n-1}^2 \cdot x_n^2 - (y_{n-1} y_n)^2 = x_{n-1}^2 \cdot x_n^2 - \left(x_{n-1}^2 + x_n^2 - \frac{1}{n^2}\right)^2 \cdot \frac{1}{n} =$$

$$= \left(x_{n-1}^2 - \frac{1}{n^2}\right)\left(x_n^2 - \frac{1}{n^2}\right) \geq 0$$

Further, let's put

$$y_1^{(1)} = y_1, \dots, y_{n-3}^{(1)} = y_{n-3}$$

$$y_{n-2}^{(1)} = \sqrt{y_{n-2}^2 + y_{n-1}^2 - \frac{1}{n^2}}, y_{n-1}^{(1)} = y_n^{(1)} = \frac{1}{n}$$

and similarly we get that

$$y_i^{(1)} \geq \frac{1}{n}, \qquad \sum_{i=1}^{n} \left(y_i^{(1)}\right)^2 = 1$$

and

$$y_1 y_2 \dots y_n \geq y_1^{(1)} y_2^{(1)} \dots y_n^{(1)}$$

Repeating this procedure $(n-1)$ times, we will eventually get the set

$$\left(y_1^{(n-1)} y_2^{(n-1)} \dots y_n^{(n-1)}\right)$$

where

$$y_1^{(n-1)} = \sqrt{\frac{n^2 - n + 1}{n}}, \qquad y_2^{(n-1)} = \dots = y_n^{(n-1)} = \frac{1}{n}$$

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and

$$y_i^{(n-1)} \geq \frac{1}{n}, \qquad \sum_{i=1}^{n} \left(y_i^{(n-1)}\right)^2 = 1$$

This means that for any set $(x_1, x_2, \ldots, x_n)$, satisfying the conditional problem, the inequality is true

$$x_1, x_2, \ldots, x_n \geq \sqrt{\frac{n^2 - n + 1}{n^n}}$$

and at

$$x_1 = \sqrt{\frac{n^2 - n + 1}{n}}, x_2 =, \ldots, x_n = \frac{1}{n}$$

equality is achieved. So the smallest value is

$$\sqrt{\frac{n^2 - n + 1}{n^n}}$$

2) Let's find the greatest value of the product $x_1, x_2, \ldots, x_n$. Applying of the theorem on averages, we get

$$x_1^2 x_2^2 \ldots x_n^2 \leq \left(\left(\frac{x_1^2 + x_2^2 + \cdots + x_n^2}{n}\right)\right)^n = \frac{1}{n^n},$$

those

$$x_1, x_2, \ldots, x_n \leq n^{-\frac{n}{2}}$$

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Equality is achieved at $x_1 = x_2 = \cdots = x_n = \frac{1}{\sqrt{n}}$

So the largest value is $n^{-\frac{n}{2}}$.

Example - 4. For given numbers $n \in N$, and $a \in [1; n]$, find the largest value of the expression $|\sum_{i=1}^{n} sin2x_i|$provided that

$$\sum_{i=1}^{n} sin2x_i = a$$

Solution. We have

$$a = \sum_{i=1}^{n} sin2x_i = \sum_{i=1}^{n} \frac{1 - cos2x_i}{2}, \sum_{i=1}^{n} cos2x_i = n - 2a$$

Next, consider vectors $(cos2x_i, sin2x_i)$

Of unit length on the plane. Their sum has length no more than n, which means the inequality holds

$|\sum_{i=1}^{n} sin2x_i| \le \sqrt{n^2 - (\sum_{i=1}^{n} cos2x_i)^2} = \sqrt{n^2 - (n-2a)^2} = 2\sqrt{a(n-a)}$, is satisfied, in which equality is achieved at $x_1 = x_2 = \cdots = x_n = arcsin\sqrt{\frac{a}{n}}$. So the largest value is $2\sqrt{a(n-a)}$.

It is easy to see that the problem of determining a conditional extremum coincides with the problem of nonlinear programming.

One way to determine conditional extremum is used if "m" variables from the relationship equations, for example $x_1 = x_2 = \cdots = x_m$, can be explicity expressed in terms of the remaining "$n - m$" variables:

$$x_i = g_i(x_{m+1}, \dots, x_m), i = 1,2, \dots, m$$

Substituting the resulting expressions for x_i into the function Z we obtain

$$Z = f(g(x_{m+1}, \dots, x_m), \dots, g_m(x_{m+1}, \dots, x_m), x_{m+1}, \dots, x_m),$$

or

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$$Z = F(x_{m+1}, \dots, x_m).$$

The problem is reduced to finding a local (global) extremum for a function of "$n - m$" variables.

Example - 5. The flour mill sells flour in two ways: retail through a store and wholesale through sales agents. When selling x_1 kg of flour through a store, sales costs are x_1^2 rubles, and when selling x_2 kg of flour through sales agents are x_2^2 rubles. Determine how many kilograms of flour should be sold in lack way so that sales costs are minimal if 5000 are allocated for sale per day kg flour.

Solution. Let's create a mathematical model of the problem. Let's find the minimum total costs

$$F(x) = x_1^2 + x_2^2 \qquad (3)$$

with restrictions

$$x_1 + x_2 = 5000, x_1 \geq 0, x_2 \geq 0 \qquad (4)$$

It is necessary to find the conditional extremum of function (3), if the connection equation has the form (4). From the equation we find, for example, x_2 ,and substitute it in (3).

$$x_2 = 5000 - x_1, F = x_1^2 + (5000 - x_1)^2 \qquad (5)$$

Wherein $x_1 \in [0; 5000]$. Let's find the global extremum of function (5) on the interval $[0; 5000]$.

The stationary point is equal to 2500, from the definition of the function we obtain that F at $x_1 = x_2 = 2500$ reaches a minimum.

Example - 6. Find the greatest value of the product $x_1^2 x_2^2 x_3^2 x_4$, provided that given number $n \geq 2$, find the largest and smallest values of

$$x_1, x_2, x_3, x_4 \geq 0 \text{ and}$$

$$2x_1 + x_1 \cdot x_2 + x_3 + x_2 \cdot x_3 \cdot x_4 = 1$$

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Solution. Using the mean theorem, we have

$$\sqrt[4]{2x_1^2x_2^2x_3^2x_4} = \sqrt[4]{2x_1 \cdot x_1x_2 \cdot x_3 \cdot x_2x_3x_4} \le \frac{2x_1 + x_1 \cdot x_2 + x_3 + x_2 \cdot x_3 \cdot x_4}{4} = \frac{1}{4}$$

those

$$x_1^2x_2^2x_3^2x_4 \le \frac{1}{512}$$

Equality is achieved if

$$2x_1 = x_1 \cdot x_2 = x_3 = x_2 \cdot x_3 \cdot x_4 = \frac{1}{4}$$

those at

$$x_1 = \frac{1}{8}, x_2 = 2, x_3 = \frac{1}{4}, x_4 = \frac{1}{2}.$$

So the largest value is $\frac{1}{512}$.

Example - 7. For given numbers $n \ge 2$ and $a > 0$, find the largest value of the sum

$$\sum_{i=1}^{n-1} x_i\, x_{i+1}$$

provided that $x_i \ge 0 \ \ (i = 1,2, \dots, n)$ and $x_1 + x_2 + \dots + x_n = a$.

Solution. Let

$$\max\{x_1, x_2, \dots, x_n\} = x_k$$

Then

$$\sum_{i=1}^{n-1} x_i\, x_{i+1} = \sum_{i=1}^{k-1} x_i\, x_{i+1} + \sum_{i=k}^{n-1} x_i\, x_{i+1} \le x_k \sum_{i=1}^{k-1} x_i + x_k.$$

$$\sum_{i=1}^{k-1} x_i = x_k(a - x_k) \le \left(\left(\frac{x_k + a - x_k}{2}\right)^2\right) = \frac{a^2}{4}.$$

Equality is achieved, for example, when

$$x_1 = x_2 = \frac{a}{2}, x_3 = \cdots = x_n = 0$$

Therefore, the largest value is $\frac{a^2}{4}$.

CONCLUSION

As a rule, in practical problems it is necessary to determine the largest and smallest values of a function in a certain area. If the area is closed and limited, then the differentiable function in this area reaches is largest and smallest values either at a stationary point or at the boundary point of the area.

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